Youth STEM Cup 2023

Final Round

Problems and Answers

29 April 2023

In collaboration with

Contents

Committee and Contributions

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Problem Selection Committee

The Problem Selection Committee (PSC) is responsible for setting and selecting problems for the contest. It makes collaborative decisions on the suitability and format of the questions, and performs cross-checks to ensure the questions are valid, clear, and well-posed. It also produces the *Problems and Answers* document, the *Problems and Solutions* document and the question paper.

Special thanks to the PSC for contributing 50 problem proposals for the Final Round:

Biology
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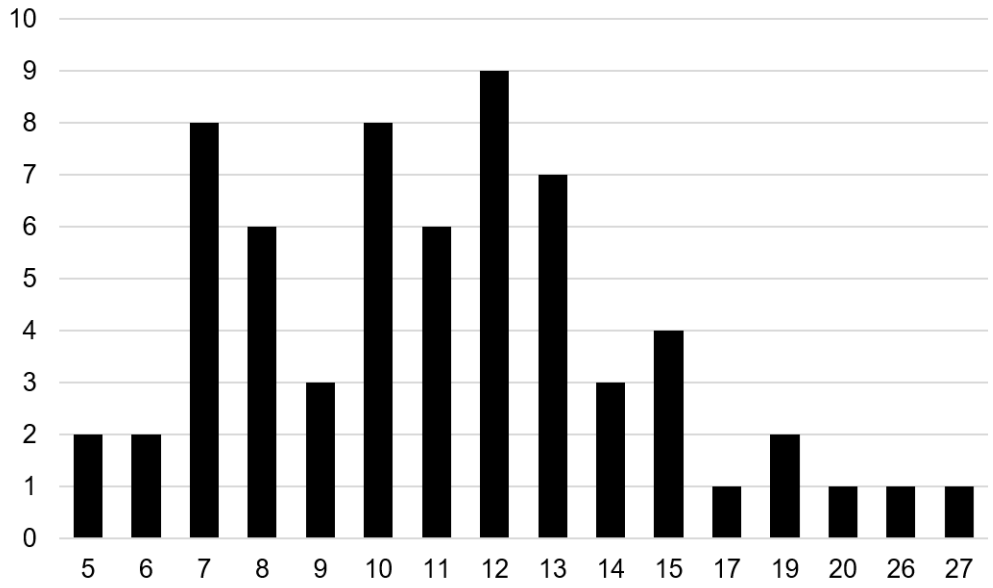
Final Round Analysis

General Statistics

Full Score	50
Number of Teams	64
Average Score	11.38
Median Score	11
Range	5-27
Standard Deviation	4.27

Score Distribution



Breakdown of Correct Responses

Question	Bio		Chem		Phy		Maths		Astro	
1-10	7	57	12	4	12	9	10	7	13	9
11-20	40	18	1	23	18	22	0	9	3	23
21-30	9	27	18	4	20	7	4	9	15	22
31-40	10	17	4	11	39	17	1	31	13	29
41-50	8	21	5	5	21	-	9	22	8	25
Σ	214		87		165		102		160	

Problems

Section A: Sky

1. Annie is the President of her School Astronomy Club. She organises a sky-gazing over the weekend and introduces her juniors to how to operate the telescope. They move all the instruments outdoors and manage to observe a few remarkable stars and the moon.

   One of the concerns for astronomers is light pollution; it reduces their ability to observe celestial objects at night. Which of the following are the effects of light pollution on us?

   I. Source of air pollution

   II. It affects the circadian rhythm of humans and animals

   III. It disturbs the migration patterns of animals

   IV. It affects animals' behaviors

   A. I, II & III B. I, III & IV C. II, III & IV D. all of the above

2. After an hour of star gazing, lightning strikes the dark sky and the team starts to pack their stuff and return to the room. Some of the juniors feel disappointed as they wish to have more time to look for more stars.

   Annie changes the mood of the team by conducting a short quiz using *Kahoot!*. One of her questions is about the importance of lightning besides as a sign of rain. One of the answer options is that lightning is involved in the nutrient cycle.

   Please arrange the following nitrogen cycle in order.

   I. Nitrifying bacteria convert ammonium to nitrate.

   II. Lightning provides energy for atmospheric nitrogen to react with oxygen to produce nitrogen oxide and nitrogen dioxide.

   III. Nitrogen-fixing bacteria convert nitrogen to ammonium.

   IV. Denitrifying bacteria strips the oxygen from nitrate and releases nitrogen back into the atmosphere.

   V. Nitrogen oxide and nitrogen dioxide enter the soil through the rain.

   A. II, III, V, I, IV B. II, V, III, I, IV C. III, II, V, I, IV D. III, V, II, I, IV

3. Annie's friend, Baneen, is a chemistry enthusiast. He tries to explain to the group how a rocket propulsion system works.

   NO_2 is a molecule with odd number of electrons. At 413 K, it can dimerise to become N_2O_4. At $T > 423$ K, NO_2 will decompose. N_2O_4 can be used as an oxidising agent to react with hydrazine (N_2H_4) in a rocket propulsion system. Write the chemical equation for the reaction within the rocket propulsion system.

4. After a series of discussions of the chemistry of various gases, Baneen also adds that some compounds will change its boiling point when mixed with other compounds, such as the well-known aromatic compounds, Benzene and Naphthalene.

   The boiling point of benzene is 80.2°C. When 2.67 g of naphthalene is dissolved in 100 g of benzene, the boiling point of this solution increases 0.531 K. What is the boiling point elevation constant of benzene? Please indicate the unit of the constant.

5. "*Why on earth are we talking about gases in an astronomy club?*" Cheng Zhen complains. He starts playing with two rubber particles in his hands.

   The two particles are released with leftward and rightward speeds v_1 and v_2 respectively. Find the time when their velocities are perpendicular.

   A. $t = \dfrac{2\sqrt{v_1 v_2}}{g}$ B. $t = \dfrac{\sqrt{2v_1 v_2}}{g}$ C. $t = \dfrac{\sqrt{v_1 v_2}}{g}$ D. $t = \dfrac{\sqrt{v_1 v_2}}{2g}$

6. Cheng Zhen then goes outside, takes one of the particles and throws it vertically upwards with the strength of the Hulk.

   When a projectile moves slowly through air, the drag is linear in the velocity, $F = -\alpha m v$. The particle is thrown upward at time $t = 0$ with speed v_0. After solving Newton's second law, we would get $v(t) = e^{-\alpha t} v_0 + \frac{g}{\alpha}(e^{-\alpha t} - 1)$. From this equation, find the terminal velocity of the particle.

   A. $-\dfrac{g}{\alpha}$ B. $\dfrac{g}{\alpha}$ C. $v_0 - (g + \alpha v_0)t$ D. $v_0 + (g + \alpha v_0)t$

Clouds have funny figures. A daydreaming (Crazy) Dave looks at a cloud that resembles a fractal, which reminds him of infinite sums. Help him solve two infinite sums:

7. Given $\alpha = \cos^{-1}(\frac{3}{5})$. Determine

$$\sum_{k \geq 0} \frac{\cos(k\alpha)}{2^k} = \frac{\cos(0)}{1} + \frac{\cos(\alpha)}{2} + \frac{\cos(2\alpha)}{4} + \frac{\cos(3\alpha)}{8} + \cdots$$

8. $\displaystyle\lim_{n \to \infty} \left(\frac{1}{2+n^2} + \frac{2}{4+n^2} + \frac{3}{6+n^2} + \cdots + \frac{n}{2n+n^2} \right) = ?$

9. "*When astronomers look at the dark sky, it is not dark but glowing with microwave radiation reminiscent of the early Universe!*" Annie explains. Dave, who has just returned from solving his maths problems, listens and puts his insanely strong imaginative skills into use:

   In an alternate Universe, astronomers discover a faint background radiation glow similar to our Cosmic Microwave Background (CMB). From their planet, they measured the glow to be stronger at one direction and fainter at the other. At the strong peak, astronomers detected that flux density peaked at $\lambda = 9.998$ cm. At the weak peak, astronomers detected that flux density peaked at $\lambda = 9.999$ cm. An astronomer hypothesised that the anisotropy (difference) is due to the motion of their star system with respect to some cosmic rest frame, where the background glow will appear uniform. Help Dave calculate the speed of their star system with respect to that frame with your knowledge of physics in our Universe.

   A. 30 cm s^{-1} B. 30 m s^{-1} C. 30 km s^{-1} D. 15 m s^{-1} E. 15 km s^{-1}

10. The celestial sphere is a projection of the night sky onto the Earth, with the celestial equator and poles matching that of Earth's equator and geographical poles. The counterparts of Longitude and Latitude on the celestial sphere is Right Ascension (RA) and Declination (DC) respectively.

    The group of students continues to stargaze after the weather has become clear. E Rick observes that the maximum altitude (angle from horizon) of Betelgeuse (an orange star in Orion) is 88° from the North side of the horizon. What is the declination of Betelgeuse?

    Note: They are in Penang (5.4164°N, 100.3327°E),

    A. $-3.4°$ B. $+2.0°$ C. $+3.4°$ D. $+7.4°$

Section B: Mountains

1. During the CNY holidays, Fatimah and her friends decides to climb Mount Kinabalu, the tallest mountain in Malaysia with a height of 4095 m from the sea level. Throughout her journey from Timpohon Gate to Laban Rata, as a naturalist, she observes various unique species of flora and fauna that she couldn't find anywhere else such as Thomas' Pygmy-squirrels and Kinabalu Shrews. These squirrels and shrews are so special from their relatives that she knew. She suddenly recalls what she has learned in her Biology class about speciation.

Figure 1: Kinabalu Shrews and Thomas' Pygmy-squirrel

Which of the following cause the speciation of these two distinct species from their ancestors?

I. Sympatric speciation
II. Allopatric speciation
III. With geographical isolation
IV. Without geographical isolation

A. I & III B. I & IV C. II & III D. II & IV

2. Fatimah also hears the ranger's explanation that there are 9 species of Nepenthaceae found in Mount Kinabalu. She realises that most of the pitcher plants are green and some are red in colour.

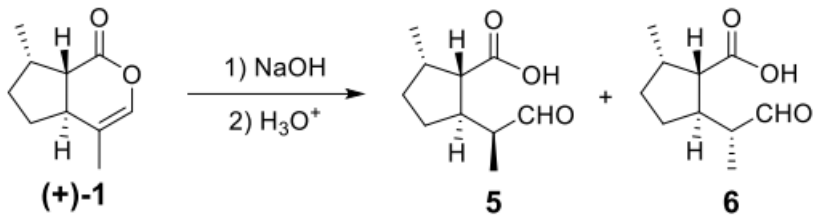
Let's say that 80% of the pitcher plants are green in colour. Estimate the number of plants that have a mixture of green and red colour if there are 100 pitcher plants. It is given that the Hardy-Weinberg Equation is $p^2 + 2pq + q^2 = 1$.

A. 4 B. 16 C. 32 D. 64

3. One of the most well-known reactions for chemists to study equilibrium is $N_2O_{4(g)} \rightarrow 2NO_{2(g)}$ The conversion percentage of N_2O_4 is 25% at P_0. Fatimah's friend, Gavin, decides to perform this experiment on Mt. Kinabalu, where the pressure is $0.7P_0$. Calculate the percentage of conversion of N_2O_4 at the peak of Mt. Kinabalu, rounded to one decimal place.
$\left(\text{Hint: } \text{Percentage}_{\text{conversion}} = \dfrac{n(\text{reactant converted})}{n(\text{initial reactant})} \times 100\%\right)$

4. Gavin also identifies a plant from the genus of Nepeta during the trip.

   Nepetalactone is one of the insect repellents produced by these plants. The reaction below shows a 2-step experiment to convert nepetalactone to compound **5** and **6**. Choose the most appropriate option on the type of reaction that occurs during the first and second step.



   A. Hydrolysis, Tautomerisation
   B. Substitution, Reduction
   C. Elimination, Oxidation
   D. Transesterification, Rearrangement

5. Seeing a ball roll down the mountain, Hoew Yang thinks of a theoretical experiment. He imagines two identically sized balls on a slope. One ball rolls down the slope while the other is smooth enough to just slide down the slope. In this experiment, which ball will reach the end first?

   A. The rotating ball.
   B. The sliding ball.
   C. Both reach at the same time.
   D. Not enough information.

6. Irene realises that the atmospheric pressure on the top of the mountain is lower than the atmospheric pressure at the base of the mountain. The difference of air pressure is caused by the the difference in molecule distribution known as Boltzmann distribution. What is the function of pressure with respect to height? It is given that P_0 is the pressure at a base height, m is the mass of a molecule, g is the gravitational field strength, h is the height of the mountain where the measurement is made, and v is the average velocity of the molecules. Note that $\exp(x)$ means e^x. (Hint: dimensional analysis)

   A. $P(h) = P_0 \exp\left(\dfrac{-mgh}{kT}\right)$
   B. $P(h) = P_0 \exp\left(\dfrac{-gh}{kT}\right)$
   C. $P(h) = P_0 \exp\left(\dfrac{-mghv}{kT}\right)$
   D. $P(h) = P_0 \exp\left(\dfrac{-mghv^2}{kT}\right)$

7. Joyton is assigned to carve a notch on a vertical cylindrical tree trunk of radius $2\sqrt{3}$ (See Figure 2) as a permanent mark of their presence. The notch is created by a horizontal plane and an inclined plane at an angle of $60°$ above the horizontal, meeting at the central axis of the trunk. Find the volume of wood removed.

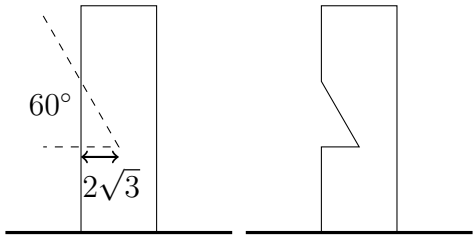


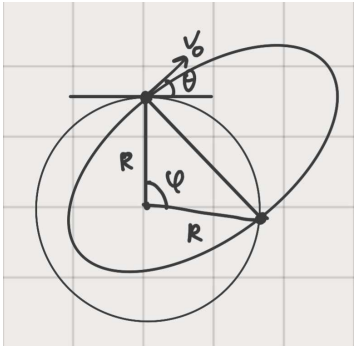
Figure 2: Tree Trunk

8. Carving a notch takes a very long time, so his maths enthusiast friend Kian Yau discusses with him a maths problem: Given two positive real numbers a, b, say a *defeats* b if $a^b > b^a$ (note that draws can happen). Determine x such that x defeats all other positive real numbers. Write `0` if no such x exists.

9. Dave is part of the group! He imagines he is designing a "space-cannon" that can shoot projectiles high up in the sky. The projectile is fired with an initial speed of $v_0 = 3$ km s^{-1}, and he wonders how far it can reach vertically. However, in his calculations, he incorrectly assumes a constant $g = 9.81$ m s^{-2}. Factoring the variation of g with height, what is the difference in the actual maximum height reached from his calculations, neglecting air resistance? (Use $G = 6.67 \times 10^{-}11$ N m^2 kg^{-2}, $R_{\text{earth}} = 6371$ km, $M_{\text{earth}} = 5.97 \times 10^{24}$ kg.)

A. 15 km B. 20 km C. 25 km D. 30 km E. 35 km

10. Dave tries to think of something else. He wants to send a package from Kinabalu to the Himalayas. In his mind, he launches the projectile with a velocity $v_0 = \sqrt{\frac{GM_E}{R_E}}$ and angle $\theta = 30°$ with respect to the horizon. By Kepler's 1st Law, the trajectory of the projectile is an ellipse (See below). How far away from Malaysia (as measured along the surface of the Earth) does the projectile travel? (Tip: conservation laws) Assume that the Earth is perfectly spherical.



A. $\frac{\pi R}{2}$

B. $\frac{2\pi R}{3}$

C. $\frac{3\pi R}{4}$

D. $\frac{4\pi R}{5}$

Section C: Laboratory

1. Kian Yau is selected to participate in the AFS Exchange Program in the United States of America for one whole semester. He is so lucky that during the program, he is allowed to visit one of the Nobel Prize winners in Chemistry to learn about the CRISPR-Cas9 genome editing technique. He and his peers are given a short quiz to check their understanding of this incredible method.

   Help him to identify which of the following is/are **CORRECT** about CRISPR-Cas9.

   I. It provides an antiviral defense system in prokaryotes

   II. It is palindromic repeats of DNA sequences found in the genomes of eukaryotes

   III. It allows the editing of genomes in vivo precisely

   IV. It allows the editing of genomes ex vivo precisely

2. Kian Yau can't believe what he sees when Jennifer Doudna, the Nobel Prize winner herself, steps inside the laboratory and welcomes the group of students. She then further explains how CRISPR-Cas9 works.

   In the CRISPR-Cas9 method, Cas9 is an enzyme that guides the target gene by forming a complex with a guide RNA with a sequence complementary to a part of the target gene. Then, Cas9 cleaves the double-stranded DNA of the target gene specifically with its activity of cleaving double-stranded DNA. Cas9 recognises a 3-base sequence (NGG) called PAM sequence and cuts the DNA strand 3 to 4 bases upstream of PAM. The cleaved DNA chain is repaired by the DNA repair system, but at that time, a few bases are frequently deleted or inserted.

   The CRISPR-Cas9 method is applied by targeting the region close to the translation start codon of the most upstream exon of a gene encoding enzyme A of a certain animal. The base sequence of the target region is determined for each of the four mutants obtained.

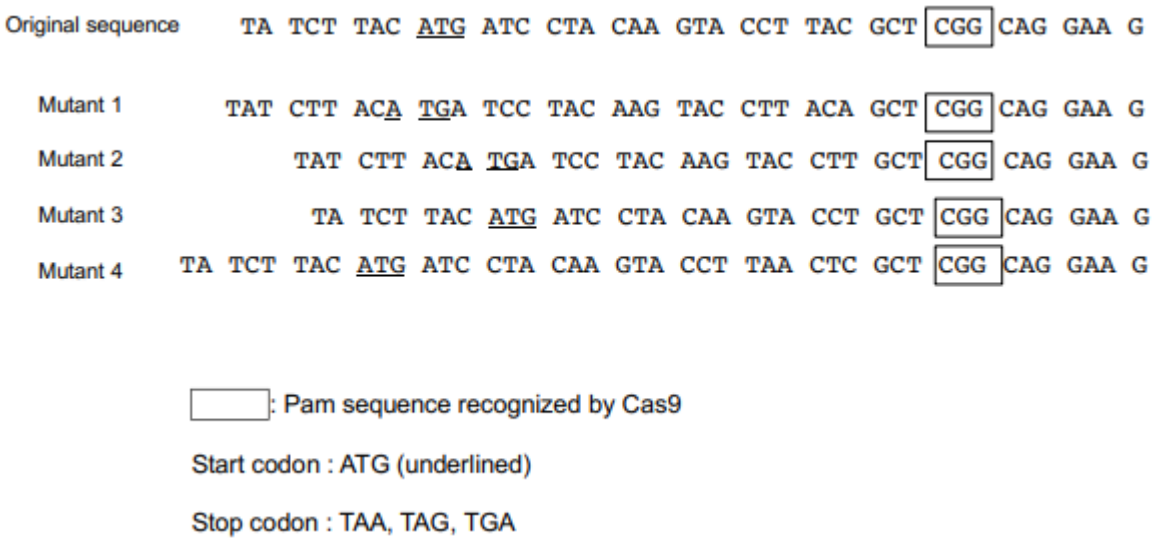


Figure 3: Source, IBO 2020

Which of the following is **TRUE** based on Figure 3?

A. It is highly likely that the activity of enzyme A is retained in mutant 1.

B. It is highly likely that the activity of enzyme A is retained in mutant 2.

C. It is possible that the activity of enzyme A is retained in mutant 3.

D. It is highly likely that the activity of enzyme A is lost in mutant 4.

After the session has ended, Kian Yau steps into a chemistry laboratory and meets Lovanovski and Mary.

3. Lovanovski is trying to find the chemical formula of an unknown compound, which he has found to be in the form KIO_x.

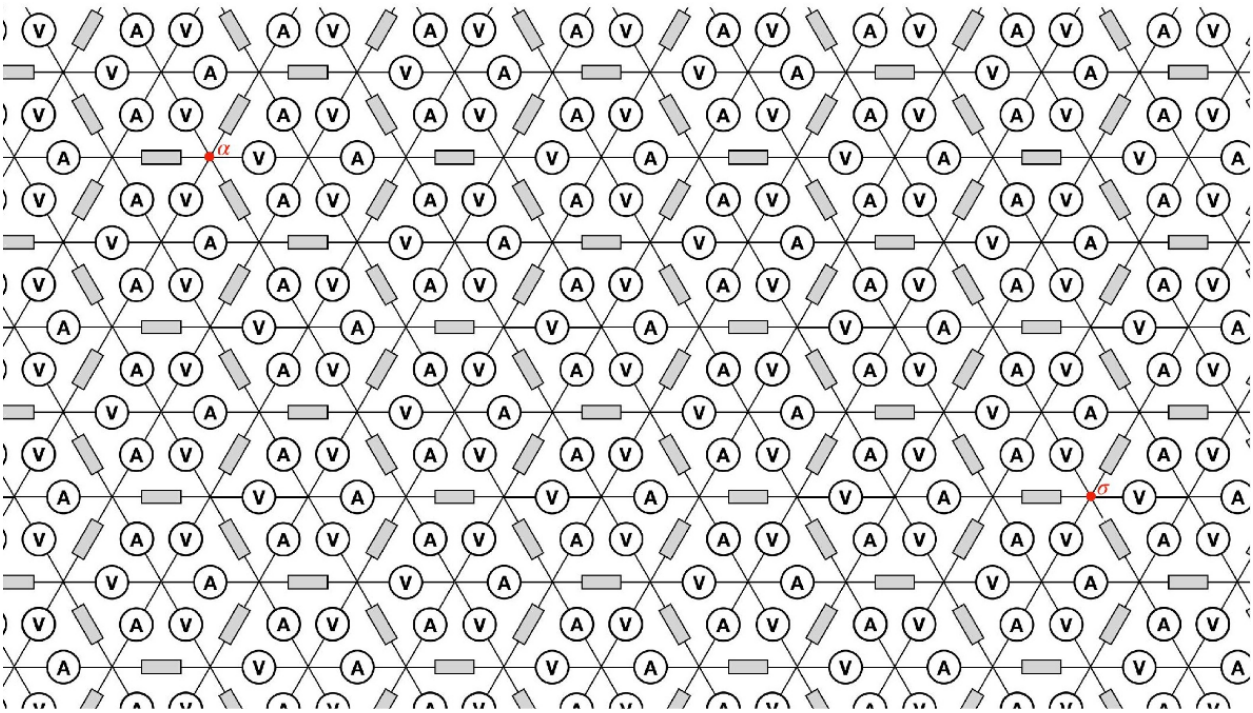
   A sample of it weighs 0.500 g. After the reduction of this sample into iodide ions, the sample is titrated against 0.1000 mol dm^{-3} of $AgNO_3$ solution. 23.36 mL of $AgNO_3$ solution is used upon reaching endpoint. What's the chemical formula of this compound? (Answer in the form of KIO_x, in which x is a whole number).

4. Mary is experimenting with aniline ($C_6H_5NH_2$), a weak base with $K_b = 4.3 \times 10^{-10}$.

   For a solution with $[Ca^{2+}] = 0.20$ M and $[C_6H_5NH_2] = 0.350$ M, what concentration of $C_6H_5NH_3^+$ is needed to prevent the precipitation of $Ca(OH)_2$, given that $K_{sp} = 6.5 \times 10^{-6}$?

Kian Yau then steps into a physics classroom. The students are solving two problems:

5. Consider an infinite network of identical and ideal electrical elements which are $R\,\Omega$ resistors, voltmeters and ammeters as shown. What is the equivalent resistance between points α and σ?



A. $\frac{1}{3}R$ B. $\frac{2}{3}R$ C. R D. $\frac{4}{3}R$

6. A point charge q is placed on the vertex of an imaginary Gaussian cubic surface. Find the electric flux through one of the faces of the cube that is non-zero.

   A. $\frac{q}{2\epsilon_0}$ B. $\frac{q}{6\epsilon_0}$ C. $\frac{q}{12\epsilon_0}$ D. $\frac{q}{24\epsilon_0}$

   The student next to Kian Yau has already finished the two physics problems and is now doing his maths homework. Help him:

7. Determine the value of $\sum_{k=0}^{505} \binom{2022}{4k} = \binom{2022}{0} + \binom{2022}{4} + \binom{2022}{8} + ... + \binom{2022}{2020}$.

8. S is a subset of $\{1, 2, 3, ..., 2023\}$ such that the product of the elements in S is not a multiple of 2023. Determine the maximum number of elements in S.

9. A breakthrough in the fusion lab granted humanity access to incredible amounts of energy!

   Back in Malaysia, The Society of Physics Students from the University of Malaya have 10^{21} joules of energy to spare. They are considering two options: either sending a rock to the Sun or expanding Earth's orbit (eg. by firing an extremely powerful planetary propulsion engine). Calculate an upper bound for the mass of the rock, and separately the maximum difference between the new Earth orbit radius and the old Earth orbit radius. The mass of the Sun is 1.989×10^{30} kg. The Earth mass is 5.972×10^{24} kg. The Earth orbit radius is 1.496×10^{11} m. Consider both the old and new Earth orbit to be circular.

   A. 7×10^{11} kg, 5 cm

   B. 7×10^{23} kg, 5 cm

   C. 7×10^{11} kg, 5 m

   D. 7×10^{11} kg, 5 mm

   E. 7×10^{23} kg, 5 mm

10. Funding for astronomy is so hard to get when you don't give government officials cool lab tours! To attract more funding opportunities, Kian Yau, back in Malaysia, decides to collaborate with the Plasma Technology Research Centre at the University of Malaya to simulate the effects of magnetic fields in threading exterior gas into a rapidly rotating molecular cloud, braking them sufficiently to collapse into stellar-size objects.

    Simulations show clouds of density 10^{-20} g cm^{-3} and radius 10^4 au would have halved its rotation speed in 10^6 years, given a magnetic field of 1 μG. In your experiment, the system is scaled down to fit in an apparatus of about 20 cm in size, and vacuum pump efficiencies constrain the cloud density to 10^{-3} kg m^{-3} to ensure the density ratio between the cloud and surrounding gas identically replicates astronomical environments. How large of a magnetic field should he provide to the system, to ensure his project shows a similar change in angular velocity within 1 year?

    A. 1 T B. 1 G C. 1 nG D. 1 mG

Section D: City

1. Natalie plans a reunion with all of her batchmates at KLCC. Her bestie suggests taking the famous Kampung Baru Nasi Lemak for breakfast. After breakfast, they walk towards the LRT station to continue their journey to KLCC. They pass by a few alleys as shortcuts to reach the station. There, they encounter a number of giant rats sneaking around the alley. Olivia recalls she has come across news before mentioning that millions of rats make KL their home. Rats thrive in every corner of the city and become the top pest infestation across the world cities including New York.

   Which of the following adaptation is/are **CORRECT** about the rat population in the city?

   I. Rats have Type II survivorship

   II. Rats have Type III survivorship

   III. Rats are K-strategists

   IV. Rats are r-strategists

   A. I & III B. I & IV C. II & III D. II & IV

2. They also notice that all the rats they see are brown rats, and none of them are black rats or grey rats. Explain why this happens.

   A. Due to natural selection where predators favor black rats over brown rats.

   B. Due to directional selection where predators favor black rats or grey rats over brown rats.

   C. Due to disruptive selection where predators favor brown rats over black rats or grey rats.

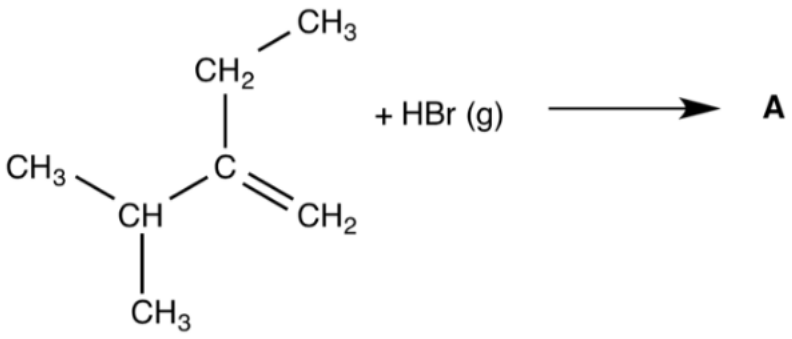
   D. Due to stabilizing selection where predators favor black and grey rats over brown rats.

3. Shifting to a different group of people on the same street, two chemists Penny and Quentin are drinking water from a dispenser on the pavement. Curious Penny talks about the entropy of ions:

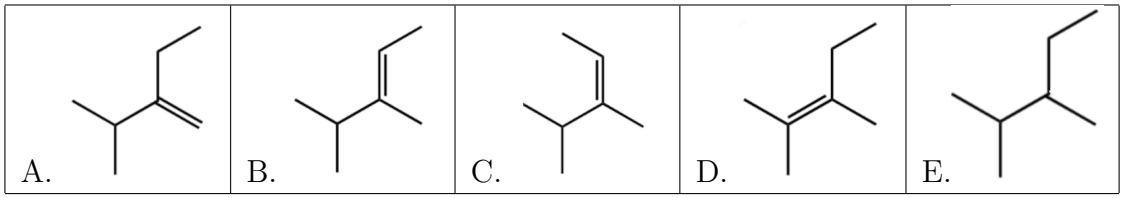
   Molecular autoionisation refers to the reaction between molecules of the same substance to produce ions. Given that the equilibrium constant for the autoionisation of water at 25°C is 1.0×10^{-14}, calculate ΔS° for $OH^-_{(aq)}$ to two significant figures using the thermodynamic data given below.

Species	ΔH_f° / kJ mol^{-1}	ΔS° / J mol^{-1} K^{-1}
$H^+_{(aq)}$	0	0
$OH^-_{(aq)}$	-229.9	?
$H_2O_{(l)}$	-285.83	69.95

4. Quentin then talks about the reaction experiment he did in the morning:



Treatment of **A** with a strong base results in the formation of several products, one of which is predominant. Which of the following shows the correct major product?



5. Riley, a very energetic batchmate of Natalie, finds herself in the middle of the road as she is playing with her phone. She is standing still on the road but then realises a truck is approaching her. She shouts as loud as she can.

   In fact, if Riley was shouting a sound with frequency f_0 and the speed of the truck was u, Riley's shouts would reflect the truck and return to her with a different frequency. Given that the speed of sound is v, what is the frequency of the waves reflected back?

   A. $f = f_0\left(\dfrac{v+u}{v+u}\right)$ B. $f = f_0\left(\dfrac{v+u}{v-u}\right)$ C. $f = f_0\left(\dfrac{v-u}{v+u}\right)$ D. $f = f_0\left(\dfrac{v-u}{v-u}\right)$

   Riley got hit by the truck and blacks out. During this period of unconsciousness, Riley finds herself in some futuristic fantasy city.

6. In this city, children don't kick balls anymore – instead, they use magnetic fields to play catch by themselves with balls of mass m and charge q in their isolated rooms. Consider the following diagram, with the magnetic field B uniformly pointing into the page:

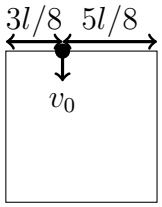


Figure 4: An Isolated Room

where l is the length of the room. Suppose the ball is given a velocity $v_0 = \dfrac{qBl}{8m}$ as shown in the figure. Determine the time required for the ball to return to its original position, assuming all collisions with the wall are elastic.

A. $t = \dfrac{\pi m}{qB}$ B. $t = \dfrac{2\pi m}{qB}$ C. $t = \dfrac{4\pi m}{qB}$ D. $t = \dfrac{16\pi m}{qB}$

7. Riley walks into a fancy restaurant where she sees 2023 couples entering too. The 4046 people plan to sit around a round table. The culture of this fantasy world dictates everyone to randomly pick a seat. Determine the expectation value of the number of couples that are seated next to each other.

8. Riley is so confused. She calms herself down with a maths problem:

   A boy has twice as many brothers as sisters. Each of his sisters has five times as many brothers as sisters. All the children have the same parents and every child is male or female.

   Assuming there is at least one sister, how many children are in the family?

9. As she is chatting with a citizen, she hears that humans are planning to build a Dyson belt on Mercury!

   The equator of Mercury has been covered with a strip of solar panels around the whole planet with a width of 5 km. The generator will have 15% power efficiency (that is, if 100 J is received then 15 J is stored). Count how many households it can power for a year after 10 days of energy capture.

   (An average household use 10000 kWh per year. The Sun has a radius of 695700 km and a surface temperature of 5778 K. Mercury is 57.9×10^6 km from the Sun and has a radius of 2440 km.)

   A. 6×10^8 B. 9×10^9 C. 8×10^8 D. 5×10^7 E. 7×10^9

10. It is also rumored that there is a rare werewolf living in the city, which locals name "The Bloodwolf", because it only appears to scare locals whenever Earth, Mars and the Moon align to form a straight line. Given the orbital period of Mars, Earth and the Moon are $T_{\text{mars}} = 686.9$ d, $T_{\text{earth}} = 365.25$ d and $T_{\text{moon}} = 27.3$ d respectively, how often does The Bloodwolf appear?

    A. 29.5 days B. 780 days C. 100 years D. 126 years

Don't worry, Riley regains her consciousness after this bizarre mind adventure.

Section E: Desert

1. As a third-year student in actuarial science, Sheldon is having a semester break and he decides to take a short course that totally deviated from his study. He finds an interesting course namely Animals at the extremes: the desert environment. He discovers that some animals that live in the desert are well adapted to remove waste without losing water by excreting concentrated urine. There are two types of nephrons that concentrate urine, a type with a short Henle loop located in the renal cortex (cortex: C) and a type with a long Henle loop located near the renal medulla (juxtamedullary: JM). The ratio of these two types of nephrons differs depending on the animal. The table shows the habitat of each animal species and the urea concentration in urine. Sheldon thinks that the camel must be the animal with the most adapted ability to preserve water in the desert. But later he finds out that he was wrong as the kangaroo rat is much more ill-adapted to strive in the desert. Table 1 below shows the comparison of urine concentration and urine/plasma (u/p) ratios in mammal species from different habitats. Figure 5 plots the juxtamedullary-cortex ratio (the number of JM type loops/the number of C type loops) in each animal species.

Table 1 (Source: `https://www.open.edu/openlearn/nature-environment`)

Mammal	**Habitat**	**Urine concentration** (mOsm/L)	**U/P ratio**
Rat	Mesic	2900	9
Domestic cat	Mesic	3100	10
Kangaroo rat	Xeric	5500	16
Beaver	Freshwater/land	520	1.7
Human	Mesic	1400	4-5
Porpoise	Marine	1800	5
Eland	Xeric	1880	6
Camel	Xeric	2800	8

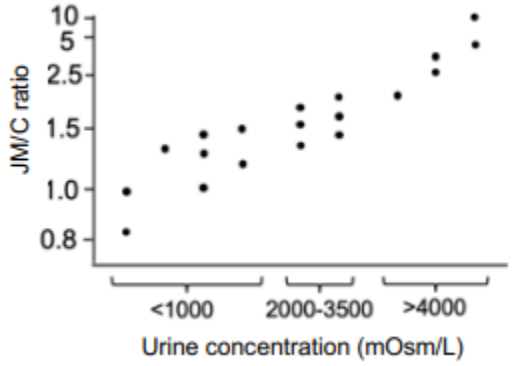


Figure 5: JM/C Ratio over Urine Concentration

Based on Table 1 and Figure 5, which of the following statement(s) is/are **TRUE**?

I. Beavers seem not to possess the cortex-type nephron

II. Kangaroo rat has the highest osmolarity of urine produced

III. Longer Henle loops can efficiently reabsorb salts, resulting in diluted urine

IV. Animals living in xeric habitats have a higher proportion of cortex-type nephrons than those living in freshwater.

2. Which of the following adaptations are found in Kangaroo rats to excrete more highly concentrated urine than camels?

I. Kangaroo rats possess extremely long loops of Henle as compared to camels

II. Kangaroo rats possess extremely short loops of Henle as compared to camels

III. Kangaroo rats possess larger medulla than camels

IV. Kangaroo rats possess larger cortex than camels

3. Sheldon's friend, Tristan, is working on the other side of the globe in the Sonoran Desert in Arizona. He has collected a sample of a mysterious mineral with a crystal structure. X-ray diffraction revealed that the unit cell of the crystal contains only one element arranged in a face-centred cubic unit structure with each side of the unit cell measuring 0.408 nm. Given that the density of the mineral is 5740 kg m^{-3}, find the relative atomic mass of this element to two decimal places.

4. Water treatment is extremely important in a populated desert. Tristan is performing a demonstration of hydrolysis to the visitors at a nearby water treatment plant.

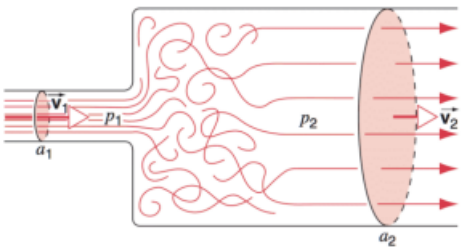
   Water is electrolysed by passing a current of 2.40 A through 1.00 L of 1.00 M K_2SO_4 solution for 2 hours. Calculate the pH of the solution in the cathode chamber after the electrolysis is stopped, given that the volume of the cathode chamber is 500 mL. ($F = 96500$ C mol^{-1})

5. One of the visitors, Ulysses, decides to play with polarising plates in the desert. Two polarising plates are stacked. The first plate has its axis at 180° to the axes of the other plate. What fraction of the intensity of an incident unpolarised beam of sunlight is transmitted by the stack?

A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. 0

6. The wind can be very strong in a desert. Ulysses's friend Vanellope thinks of an experiment.

   Consider a stream of fluid of density ρ with speed v_1, passing abruptly from a cylindrical pipe of cross-sectional area a_1 into a wider cylindrical pipe of cross-sectional area a_2 as shown. The jet will mix with the surrounding fluid and, after the turbulent mixing, will flow on almost uniformly with an average speed v_2. Which of the following options is **WRONG**?



   A. $a_1 v_2 = a_2 v_1$

   B. $p_1 - p_2 = \rho v_2 (v_1 - v_2)$

   C. $a_1 v_1 = a_2 v_2$

   D. $p_1 - p_2 = \frac{1}{2}\rho(v_1^2 - v_2^2)$

7. Wei Yan unfortunately loses their tourist group and finds herself in the middle of nowhere. Before all hope is lost, a genie springs up from the sand and is kind enough to guide her through the desert. However, in return she has to make an effort to help the genie with his homework from genie school: Find the units digit of

$$2023^0 + 2023^1 + 2023^2 + \cdots + 2023^{2023}$$

8. As the sky becomes dark, Wei Yan and the genie have to seek shelter to avoid the strong breeze at night. The genie is capable of generating a fence but has energy to only conjure one with perimeter at most 8. And, for some reason, the genie likes eight-sided figures. What is the maximal area of a octagon with perimeter 8?

   Wei Yan successfully leaves the desert the next morning. She bids goodbye to the genie.

9. The genie turns out to be a manifestation of foreign cosmic energy found from a meteorite crater in the desert. From the crater, a team of first-responders led by Prof. Xavier discover that an alien civilisation has delivered a package: A table that includes physical quantities in alien units. The aliens referenced their host star to have an alien-magnitude of 10 when observed at 1 alien-distance away, and alien-magnitude of 9 when observed at 4 alien-distance away. Linguistic expert Yusuf deciphers that the alien-magnitudes are reported in base-2 logarithm scale (recall that our brightness magnitudes are reported in base-10 logarithm scale) and the alien-distance is reported linearly, just like our distances, with 16 pc equal to 1 alien-distance. In the astronomers' records, they find the alien host star to have an absolute magnitude of 5 and is located 128 pc away from Earth. What is the observed brightness of the alien host star as reported in alien-magnitude and (normal) Pogson's magnitude? Hint: the difference of alien-magnitudes is proportional to the base-2 logarithm of the ratio of brightness.

   A. 7.5, 10.5 B. 8.5, 10.5 C. 9.5, 10.5 D. 9.5, 7.5 E. 10.5, 7.5

10. Amidst the dusty atmosphere caused by the meteor impact, one of the observers, Zhi Zheng, notices that one speck of dust on the sky seems to be fixed - a blurry object... a galaxy! He knows that historically, one of the earliest predictions for Dark Matter came from analyzing the rotation curves of galaxies, i.e. by looking at $v(r)$ against r where r is the distance to the galactic center. Here, Zhi Zheng derives an expression for $v(r)$ as a function of r by approximating the Milky Way as a cylinder of height z_0, radius R_0 and density which is ρ_0 inside and zero outside.

    However, the observed velocity curve by astronomers is $v_{\text{obs}} = \sqrt{G\pi\rho_0 z_0 R_0}\left(\dfrac{5/2}{1+e^{-4r/R_0}} - \dfrac{5}{4}\right)$. Because of this discrepancy, astrophysicists theorise the existence of Dark Matter, to make up for the 'missing matter' that makes the galaxy spin. How does the extra mass, $M_{DM}(r)$, scale with r for $r < R_0$?

    A. $M_{DM} = \pi\rho_0 z_0 R_0^2 \left(\dfrac{5/2}{1+e^{-4r/R_0}} - \dfrac{5}{4}\right)^2$

    B. $M_{DM} = \pi\rho_0 z_0 R_0 \left[r\left(\dfrac{5/2}{1+e^{-4r/R_0}} - \dfrac{5}{4}\right)^2 - R_0\right]$

    C. $M_{DM} = \pi\rho_0 z_0 R_0 \left[r\left(\dfrac{5/2}{1+e^{-4r/R_0}} - \dfrac{5}{4}\right)^2 - \dfrac{r^2}{R_0}\right]$

    D. $M_{DM} = \pi\rho_0 z_0 R_0 \left[r\left(\dfrac{5/2}{1+e^{-4r/R_0}} - \dfrac{5}{4}\right)^2 - r\right]$

Answers

Section A

1. D
2. B
3. $2N_2H_4 + N_2O_4 \rightarrow 3N_2 + 4H_2O$
4. 2.54 K mol^{-1} kg
5. C
6. A
7. 14/13
8. 0.5
9. E
10. D

Section B

1. C
2. C
3. 29.5%
4. A
5. B
6. A
7. 48
8. e(=2.718...)
9. E
10. B

Section C

1. I,III
2. C
3. KIO_3
4. 2.64×10^{-8}
5. B
6. D
7. 2^{2020}
8. 1905
9. A
10. C

Section D

1. B
2. D
3. -10.6 J mol^{-1} K^{-1}
4. D
5. B
6. D
7. 4046/4045
8. 7
9. C
10. D

Section E

1. II,III
2. I,III
3. 58.69
4. 13.55
5. C
6. -
7. 0
8. $2 + 2\sqrt{2}$
9. B
10. C